Putting Economics (Back) into Quantitative Models

The economic state of the world does matter.

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Quantitative models are assumed to operate in a theoretically ideal environment. A physicist modeling the behavior of an elephant in the jungle makes the assumption that the elephant can be thought of as a large sphere. Everything that follows then depends on this assumption, so that the model can be expected to fail under a wide range of conditions when the elephant is not really a sphere.

Myron Scholes [2005] should know:

> We make models to abstract reality. But there is a meta-model beyond the model that assures us that the model will eventually fail. Models fail because they fail to incorporate the inter-relationships that exist in the real world.

I argue that incorporating economic principles, such as demand and supply, investor behavior, and preferences from microeconomics, and monetary policy, macro aggregates, deficits, and trade balances from macroeconomics, can help make our models more flexible, and hence more robust—and ultimately more useful for making and retaining profits. My examples are from the fixed-income markets, but I suspect that many general principles have parallels in other markets.

Recent research illustrates that we can build better models without giving up the basic principles that form the foundations of arbitrage-free pricing; see Ang and Piazzesi [2003], Dewachter and Lyrio [2002], Diebold, Piazzesi, and Rudebusch [2005], Diebold, Rudebusch, and Aruoba [2003], Evans and Marshall [2001], Hördahl, Tristani, and Vestin [2002], Piazzesi [2003], Rudebusch [2003], and Wu [2001]).

Using economics as the backdrop enables "better estimates of the distribution of future outcomes than a simplistic stochastic process of an unobserved factor estimated over a period of decades" (Sack [2005]). Models fail not because the math underlying them is wrong, but because by blindly going to the risk-neutral framework we often misprice the risk premium, and hence misprice risk.

Risk-neutral valuation is a powerful computational tool—because it implies that if we can replicate the movement of any security into continuously tradable replicating portfolios, and actually trade the replicating portfolios continuously, we may choose any probability measure we want. It's the "actual" part that causes problems and the conflict between theory and reality. There is simply no instantaneousness or continuity in real markets. Economics, risk aversion, and preferences creep in. Successful investors' models explicitly incorporate the economic environment *ab initio* without throwing away any of the arbitrage-free framework.

To take stock of the main reasons and the shortcomings of the models with which we have experience, I discuss their relevance for making money, model solvability, arbitrage considerations and the irrelevance of investor behavior, and preference and security prices.

RELEVANCE FOR MAKING MONEY AND MANAGING RISK

There are four ways to generate honest investment returns:

- Taking factor risk exposures (macro).
- Intermediation (brokering).
- Liquidity and or risk transfer (insurance).
- Mispricing (arbitraging inefficiency).

Most investment activity represents combinations of these modes. The economic environment is the backdrop on which appropriate models are calibrated. Whether used for taking factor exposures, intermediating, warehousing or arbitraging, option pricing model development typically works in the sequence as follows:

1. Assume a distribution for the fundamental variables (for the standard LIBOR market model, forward rates are assumed lognormally distributed), and estimate the parameters using history and common sense as input.
2. Generate probable paths of evolution.
3. Fit remaining free parameters in the model to traded security prices.
4. Price other securities with the model.

This framework is consistent for relative pricing, but because of underlying distributional assumptions it is not equipped to adapt automatically as the underlying dynamics undergo a structural shift. Corrections must be incorporated by hand to ensure that the model remains well-calibrated, which is not always viable. Given their inability to adapt, traditional lognormal models had to be abandoned for pricing and trading 0% floors as Japanese nominal rates dropped to zero over the last 15 years.

A model that works well for long-term investing and is designed to warehouse risks fails miserably if it is used for high-frequency directional trading or arbitrage, and vice versa. Dealer desks make money through the flow business—they need only models that can be calibrated locally to today's market, because tomorrow the position will be unloaded to another customer. For short holding periods, it does not make a big difference if the model is calibrated improperly to the big picture. Levered arbitrage hedge funds make money by taking residual risks and need models properly calibrated to recurrent episodes of vanishing liquidity and deleveraging.

Mutual funds and insurance companies usually do not lever, so their risk-taking depends on the ability to hold illiquid positions for longer periods. Hedge funds and mutual funds frequently operate in the area of illiquid or distressed assets, but differ fundamentally in their ability to obtain funds, and this key difference has to be made explicit in the risk models they use because the difference in financing impacts holding-period returns. Between the time of putting a position on and taking it off, the world can change; model recalibration shocks should not force an unwinding of positions. While dealer desks may work within the risk-neutral framework, mutual fund and and hedge fund investment decisions are better evaluated using versions of models based on real-world measures.

STABILITY, ANALYTICAL TRACTABILITY, AND THE ABILITY TO STRESS-TEST

Exact solvability has forced many term structure and credit modelers to confine their attention to a subclass of affine models. With a limited set of prices of bonds and swaps as inputs, but not their options as inputs, affine models do not easily replicate the underlying dynamics of the yield curve and its volatility very well (see Collin-Dufresne, Goldstein, and Jones [2004]).

Good models would have yield curves and volatilities described simultaneously since practitioners know that the shape of the yield curve also reflects premium due to uncertainty. If we are willing to take the tractability of affine models as a requirement, they should excel in compensation through more economic content.

Even if we could surmount the technical limitations of affine models using more sophisticated albeit numerical models, the assumption of continuity in a world full of structural breaks is harder to get around. Most models lean very heavily on the notion of stability by assuming stable distributions for the dynamics, which, after a hundred years of stochastic calculus, are practically all we have to work with. But markets and policies undergo rapid structural breaks. Since most term structure models are based on the principle that long rates are time averages of short rates plus a risk premium, there is no way to account for the possibility of structural breaks unless that is built in from the very beginning in the policy setting function.

To illustrate this further, let's discuss the basic building block for interest rate models—the short rate. The Taylor [1993] rule, well-regarded as the monetary policy response function, for the funds rate is:

$$i_t = (r^* - \theta_1\pi^*) + \theta_1(\pi_t) + \theta_2(u^* - u_t) + \pi_t \quad (1)$$

where i_t is the short rate, π_t is the inflation rate, and u_t is the unemployment rate at time t; r^* is the equilibrium real rate; and π^* and u^* are the target inflation and unemployment rates. To infer the trade-off between unemployment and inflation gaps for a policy setting central bank, θ_1 and θ_2 are coefficients typically estimated by linear regressions. Yields as a function of maturity can be obtained by integrating the time evolution of the short rate i_t and computing the log expectation.

Fed behavior in setting monetary policy has exemplified major structural breaks (e.g., when transitioning from the pre-Volcker era to the Volcker-Greenspan era; see Clarida, Galí, and Gertler [2000] and Judd and Rudebusch [1998]) and perhaps more recently with Long-Term Capital Management in 1998.

Exhibit 1 illustrates the great variation in the estimates of the constant (i.e., the relation between the equilibrium real rate and the target inflation rate) as well as the weights on inflation and growth during the tenure of different Fed chairmen. Exhibit 2 picks out structural breaks using a simple recursive residuals test for breaks (Greene [2005]). Falling inflation rates and inflation volatility displayed in Exhibit 3 strongly indicates a change in underlying economic variables.

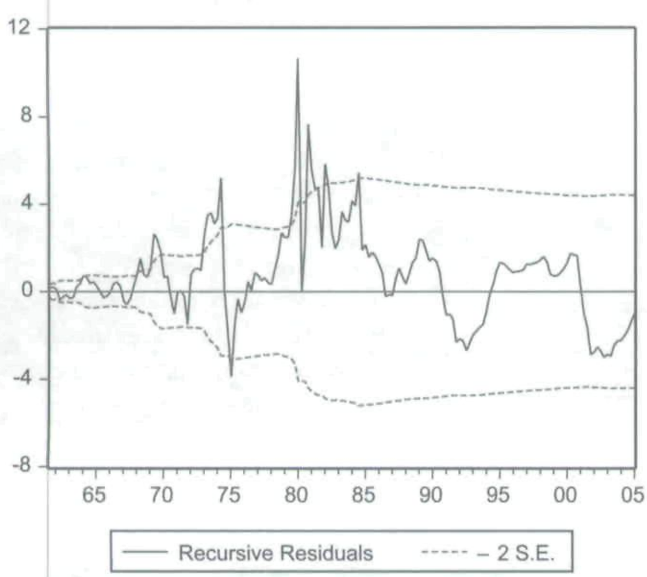
Switch to credit modeling, which is all the rage in academia and industry today. The current state of the art assumes a copula function to describe the joint behavior of loss; quants know the limitation of the

EXHIBIT 1
Estimation of Relation Between Target Real Rate and Inflation Rate

Period	c	θ_1	θ_2	R^2
Burns (1970Q3-1978Q1)	-2.118 (1.835)	0.555 (0.326)	2.737 (0.483)	0.54
Volcker (1979Q3-1987Q2)	1.295 (1.120)	0.561 (0.178)	0.623 (0.280)	0.72
Greenspan (1987Q3-2005Q1)	-0.435 (0.295)	1.107 (0.110)	1.995 (0.128)	0.859
Greenspan (1987Q3-1998Q3)	1.109 (0.188)	0.540 (0.059)	1.940 (0.069)	0.96
Greenspan (1998Q4-2005Q1)	1.379 (0.546)	0.161 (0.339)	2.494 (0.093)	0.96
(1960Q1-2005Q1)	1.279 (0.372)	0.330 (0.090)	0.562 (0.136)	0.58

Relation for different federal reserve regimes using Equation (1). The constant c is given by the Relation $c = r^* - \theta_1\pi^*$. *Standard errors are in parentheses.*

EXHIBIT 2
Recursive Residuals



Demonstrates possibility of structural breaks in Taylor rule in the mid-1970s-mid-1980s.
Source: PIMCO.

approach, but as of today there is no common benchmark model to improve upon the gross shortcomings of the copula approach. The Gaussian copula is to structured credit what the lognormal assumption is to option valuation, the Black-Scholes of options on portfolio losses, tractable but not too accurate except for use by dealer desks.

The base correlation skew of copula-based models is akin to the volatility skew of options. The key simplification in the copula approach is provided by the assumption of a single input default correlation parameter that is not directly observable. When Northwest and Delta entered bankrupty within minutes of one another in September 2005, we saw default correlation in practice, and it was 1. Without ex ante input we cannot statistically estimate this number from data.

To describe the reason for the simultaneous defaults, most analysts would fall back on macroeconomic variables as the cause—e.g., underfunded pensions, falling revenues, competition, increased oil prices. But it seems hard, if not impossible, to include macroeconomic inputs as complex as these in one correlation parameter. Nonetheless, we settle by fitting market prices to imply the value of the parameters and then using the parameters for valuation on increasingly more complex and non-linear securities.

We even trade tranches assuming that they can be packaged together in an essentially arbitrage-free way, which in incomplete markets of the day is just a wrong assumption (recovery products are not liquid currently). No wonder that tranches do not behave according to model.

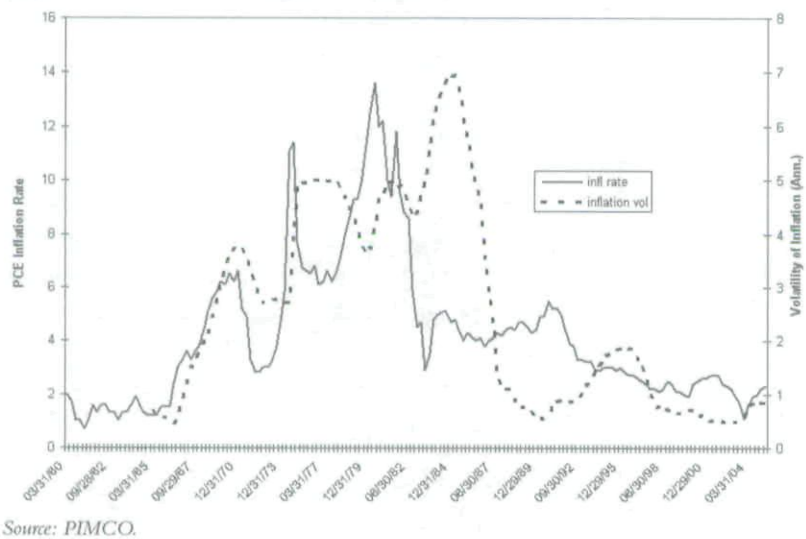
Exhibit 4 shows the actual and realized delta-adjusted return of tranches in the spring of 2005 from a dealer's post-mortem report on tranche performance. Not only was the hedge not a hedge in size, but for many tranches the sign was also wrong.

At the very least, if the copula model for trading default correlation has any value, we should be able to attribute proportions of the default probability to economic factors. In other words, if the model is required to possess any depth, what is exogenous has to be made endogenous. Some recent progress has been made in this direction without giving up analytical tractability that allocates the default risk to economywide, sector, and idiosyncratic risks (Duffie and Garleanu [2001], Longstaff and Rajan [2006]).

The limitations of using risk-neutral pricing become all too obvious in stressed markets. Hedge ratios take on a life of their own; to hedge, underlying securities are traded. When the market hedge ratios are high enough, the act of dynamic hedging can influence the price of hedging securities in an amplifying feedback. When there is enough of an impact, model-driven hedges can actually lead to systemic market distortions, such as the numerous mortgage debacles of the last decade. Strangely, we allow for "slippage" in hedging, but not in valuation.

Anecdotally, the rapid rise of quantitative credit models has coincided with numerous

EXHIBIT 3
Inflation Rate and Inflation Volatility



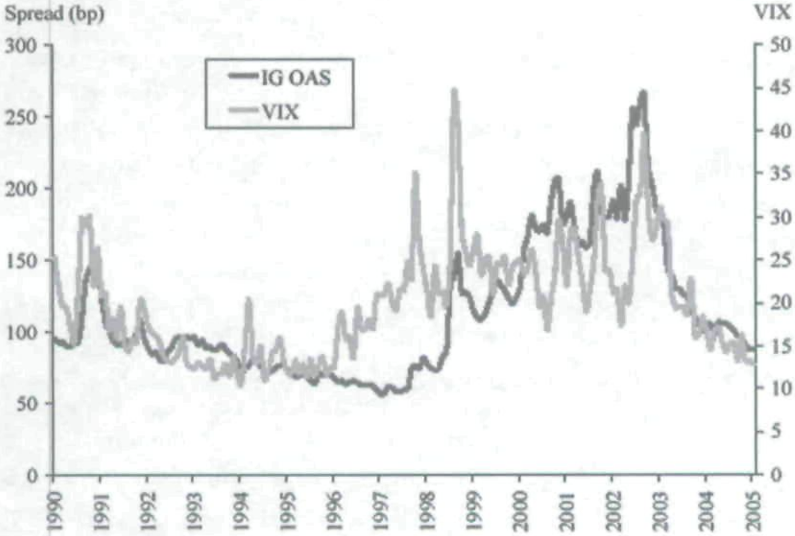
Source: PIMCO.

EXHIBIT 4
Delta-Hedged Tranche Price Changes in May (bp)

	5Y		10Y	
	US	Euro	US	Euro
0-3% (points upfront chg)	4.0pts	3.9pts	1.5pts	-2.5pts
3-7%	(84)	(49)	133	28
7-10%	(38)	(24)	(85)	(19)
10-15%	(15)	(11)	(37)	(55)
15-30%	(2)	(1)	(14)	(6)

Delta-hedged positions involve selling protection on the tranche versus buying protection a delta-adjusted notional amount of the underlying index.
Source: Morgan Stanley.

EXHIBIT 5
VIX and Corporate Spreads



Source: Lehman Brothers.

EXHIBIT 6
FNMA 5.0% 30-Year Pass-Through Option-Adjusted Spreads and Option-Adjusted Durations

Dealer	Treasury OAS	Libor OAS	OAD (Research)	OAD (Trading Desk)
Lehman	26	3	4.8	4.6
Goldman	81	34	4.6	4.0
Greenwich Capital	36	17	4.0	4.0
CSFB	52	18	6.1	4.4
Salomon	51	16	5.2	4.4
Morgan Stanley	52	29	5.0	4.2
Bank of America	64	25	4.2	4.5
UBS	44	20	4.9	4.7
Countrywide	107	50	5.1	4.1
JP Morgan	54	23	6.1	4.1
Merrill	52	21	5.0	4.1
Bear	65	21	5.3	4.3
Average	57	23	5.0	4.3
Range	81	47	2.1	0.6
Min	26	3	4.0	4.0
Max	107	50	6.1	4.7

Figures as of May 05, 2003.
Source: PIMCO survey.

proprietary "capital structure arb" desks and dedicated hedge funds. Applying a Merton modeling framework to credit and equity, a new theoretical connection has become all too easy to impose. Is falling equity volatility that has been accompanied with tightening credit spreads as in Exhibit 5 a coincidence or a consequence of massive-scale application of these sorts of models?

Tractable value at risk models are notorious for failing when you need them most and for working great when you don't need them. Most VaR models are based on historically estimated covariance matrices, so they cannot see stresses unless the underlying covariance matrix is taken to its mathematically extreme values. To take covariance matrices systematically to logical extreme values, economic priors and portfolio risks under those outcomes have to be built in right from the beginning (Bhansali and Wise [2001]).

A VaR model based on history alone would almost never capture episodes like bull flattening, where yields and the steepness of the curve drop simultaneously. As many mortgage investors recently found out, valuation and risk models did very badly in the 2004-2005 bull flattening episode.

Are we chasing hurricanes in Canada? Not unimaginable if we allow for parameters to take values under structural shocks that might not have occurred in the past but are probable nonetheless.*

FREEDOM FROM ARBITRAGE

State of the art models are approximations that largely ignore demand and supply considerations. Armed with the best arbitrage-free quantitative models possible, experts can and do disagree, as we will seebelow.

Exhibit 6 shows the wide variation in the option-adjusted spread and duration numbers from a survey of the top mortgage-backed securities dealers on Wall Street for the same security on a given day. A levered derivative structure like an interest-only instrument would magnify these errors even further. Combining the principal-only and the IO from a liquid pass-through still allows for a non-zero total risk premium due to a persistent type of risk—the potential for model error that cannot be arbitraged away. This risk premium, not surprisingly, tends to rise when there is more economic uncertainty.

What also stands out is that the research desks and trading desks don't usually agree—the trading desk effectively adds a risk correction to the research models. Since risk-takers are the ones who have the most to lose from bad models, they supplement the research models with their economic priors by applying a haircut to the model numbers, and make the dispersion less to match the market.

For fixed-income instruments, we can think of any yield over a benchmark instrument as arising from an embedded option. Even yield enhancement from duration extension in upward-sloping yield curves can be thought of as the price for an option to reinvest at a higher rate than that implied in the forward curve. Investors do not all value options efficiently, and, even if they did, under different economic conditions they have neither the will nor the means to exercise these options in the optimal fashion. This can lead to what appear to be grossly inefficient markets from the viewpoint of arbitrage-free models unless principles of economics—demand and supply—are imposed endogenously.

A striking example of the bounds of arbitrage comes from the Treasury futures market, which is one of the largest and most liquid derivatives markets anywhere. When the contract was designed almost 20 years ago, open interest in the contract was expected to be comparable to the size of the cheapest to deliver securities. Thus one could use arbitrage-free derivatives models to value the delivery option and trade the cash versus futures basis. As time has passed, the ease of use of futures has increased participation in the futures contract, while the size of the cash Treasury bond market has dwindled (for various reasons, including the start of a Treasury buy-back program in 1999-2000).

The mismatch in the size of the deliverables to the open interest in the contract (approximately $20 billion in the cheapest to deliver U.S. Treasury of 4.375% coupon maturing in August 2012 versus approximately $160 billion in the December 2005 futures contract) has made application of arbitrage-free theoretical models largely irrelevant for rich-cheap valuation. In 2005 the contract traded at once unheard of rich levels, as there was a mad scramble to procure the then-current cheapest to deliver.

INVESTOR BEHAVIOR AND PREFERENCE IS IRRELEVANT TO THE MODEL

Risk premium in incomplete markets cannot be hedged, and unhedgeable risk premium comes with unhedgeable risk. This creates value for someone who has the room to take that risk. Security prices reflect the price of risk transfer, and in the real world, investors do not have identical preferences, so it is impossible that all securities can be modeled without taking investor risk preferences and behavior into consideration.

As we have noted, however, most pricing taught in schools assumes that investors can always be made locally risk-neutral. The market is assumed to be complete, and risk aversion is assumed to be irrelevant because one can always transform to the risk-neutral measure. Large unhedgeable moves in the market are routine, and even the best structured relative-value trades expose their absolute risks and prove the falsity of this assumption. Locally hedgeable portfolios are the exception rather than the rule (forget about globally hedgeable portfolios; the only way to create them is to be a broker and lay off the risk completely or sell your position).

Theoretically a risk premium is proportional to the covariance of the investor's utility with the economic state of the world (Cochrane [2005]). In other words, assets that lose their value when you are poor suffer a penalty (since when you are poor you value the marginal dollar more), and assets that gain in value in bad states of the world carry a premium (such as Treasury bonds). In practice, most consumers probably do not carry out the life-cycle consumption maximization exercise, but rather look at the economy they can collectively observe and expect.

Market volatility has dropped to low levels, but so has the volatility of economic variables such as inflation and GDP. The Fed now also moves in smaller steps.

To understand the sources of risk, and the risk premium, it is essential to understand the economics of the market environment. Asset classes can quickly influence each other and lead to simultaneously lower risk-adjusted returns across the cross-section to reflect macroeconomic realities.

In Exhibit 7 we see that over five-year horizons the duration-neutral excess returns and information ratios of various fixed-income assets show a substantial directional bias correlated with the level of the bond market overall. Assuming that assets will covary according to stable historical correlations in all environments is a dangerous

Exhibit 7

Asset Excess Return Performance and Correlation to overall bond market as represented by the Lehman Brothers Aggregate

Asset Return Comparisons (Rebalanced to LBAG Duration Monthly, LIBOR Financing)										(07/91	07/05)		
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
MTD	(0.91)	(1.17)	(1.00)	(1.01)	(0.63)	(1.16)	(1.12)	(0.17)	(0.06)	1.59	(1.58)	(1.38)	(1.25)	(0.91)
YTD	1.58	1.09	1.93	1.46	1.60	1.20	1.01	2.17	3.91	2.50	(0.97)	0.31	1.68	3.25
0.25	0.71	0.30	0.59	0.59	1.13	0.49	(0.26)	0.86	3.40	5.06	(0.18)	(0.10)	0.36	1.10
0.5	0.95	0.39	1.29	1.03	0.94	0.66	0.98	1.53	3.39	2.61	(0.51)	0.39	1.26	2.15
1	4.79	4.66	4.54	4.25	5.27	4.58	5.07	4.81	12.82	10.43	0.96	3.14	4.72	6.61
2	4.81	5.21	4.06	4.26	5.20	4.73	5.97	4.31	11.33	11.48	2.14	4.37	5.00	5.42
3	5.02	4.10	3.96	4.28	6.14	5.07	6.10	3.74	14.95	14.85	4.58	4.84	4.82	4.60
5	7.01	7.27	5.90	7.13	7.08	8.16	7.93	4.93	9.55	6.58	10.62	8.21	7.26	5.53
7	6.21	6.23	5.59	6.18	6.11		7.38	4.75	9.60	4.43	8.12	7.27	6.70	5.42
10	6.75	6.92	6.25	6.75	6.69						8.07	7.48	7.22	6.32
14	7.29	7.24	6.89	7.26	7.46						9.15	8.28	7.85	6.78
Information Ratio Comparison (Annualized Excess Return over LBAG/Annualized Standard Deviation of Excess Return, monthly data)														
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
0.5	0.00	(1.82)	1.37	0.62	(0.01)	(1.28)	0.05	0.72	1.55	0.46	(2.37)	(1.01)	0.61	5.60
1	0.00	(0.21)	(0.52)	(1.29)	0.72	(0.29)	0.19	0.01	3.03	1.06	(1.98)	(1.32)	(0.08)	2.87
2	0.00	0.44	(1.56)	(1.31)	0.66	(0.10)	0.60	(0.27)	2.18	1.43	(1.04)	(0.26)	0.21	0.77
3	0.00	(0.30)	(1.55)	(1.36)	1.32	0.05	0.52	(0.62)	2.02	1.64	(0.16)	(0.09)	(0.15)	(0.50)
5	0.00	0.10	(1.35)	0.14	0.05	1.04	0.44	(1.01)	0.34	(0.05)	1.07	0.56	0.17	(1.30)
7	0.00	0.01	(0.65)	(0.03)	(0.07)		0.50	(0.68)	0.22	(0.20)	0.53	0.50	0.31	(0.59)
10	0.00	0.09	(0.62)	0.00	(0.05)						0.42	0.39	0.34	(0.36)
14	0.00	(0.03)	(0.55)	(0.06)	0.17						0.57	0.54	0.42	(0.43)
Excess Return Correlation vs. LBAG														
Date	LBAG	MBS	TSY	AGCY	CORP	CMBS	TIPS	Muni	EM	HY	ED5	FV	TY	US
0.5	1.00	0.45	0.18	0.43	(0.35)	0.93	(0.55)	(0.94)	0.03	(0.44)	0.94	0.51	0.54	0.43
1	1.00	0.10	0.07	0.34	(0.22)	0.74	(0.47)	(0.95)	0.08	(0.43)	0.74	0.58	0.52	(0.10)
2	1.00	0.32	(0.30)	0.48	(0.28)	0.73	(0.41)	(0.92)	0.24	(0.45)	0.77	0.75	0.73	(0.48)
3	1.00	0.39	(0.32)	0.28	(0.27)	0.59	(0.18)	(0.93)	(0.02)	(0.46)	0.38	0.56	0.62	(0.21)
5	1.00	0.42	(0.13)	0.36	(0.33)	0.49	(0.24)	(0.87)	(0.20)	(0.38)	0.51	0.55	0.58	(0.17)
7	1.00	0.38	(0.14)	0.27	(0.26)		(0.32)	(0.82)	(0.13)	(0.38)	0.47	0.51	0.52	(0.18)
10	1.00	0.32	(0.11)	0.24	(0.22)						0.44	0.52	0.55	(0.09)
14	1.00	0.20	(0.07)	0.14	(0.12)						0.53	0.57	0.59	(0.13)

Source: PIMCO.

assumption—it throws out the experience that asset performance comes eventually from the need for the asset in an investor's full portfolio, not from a stand-alone pure value.

Portfolio needs and mixes of assets change as the economic environment changes. Regulations, the other side of economics, may bias security risks and performance as well. Many marginal investors are prohibited from ownership of companies but not from lending to them, and cash has gone preferentially into corporate bonds rather than equities since the 2000 equity market meltdown. Using Merton models to relate the structural approach to the reduced-form approach requires paying heed to this.

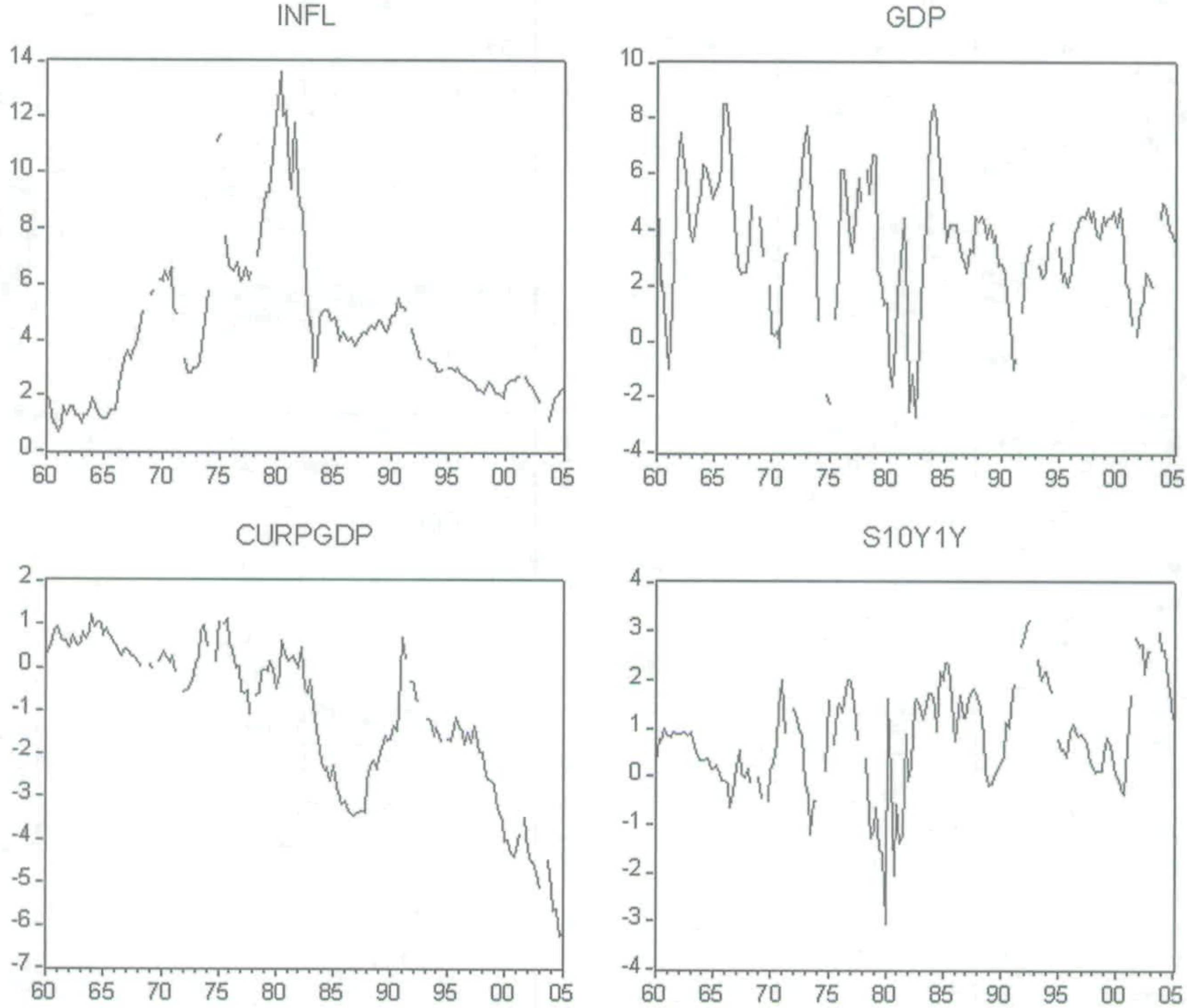
THE MODELER SHOULD BE IRRELEVANT TO THE MODEL

The impact of the modeler's human capital is assumed to be largely irrelevant. Modelers take short-cuts, make approximations, and follow logic unique to themselves. The steps are frequently not commutative, and reordering yields significantly different conclusions.

It is no secret that in recent years the single greatest influence on fixed-income markets has been the growth of mortgages as an asset class, and the commoditization of prepayment risk. Since the right to prepay is an option for the homeowner, the investor in mortgages is short the prepay option.

Exhibit 8

Macroeconomic Data Series for Fitting Two-Factor Model



Source: PIMCO.

To figure out how much interest rate risk can change as prepayments change, modelers build models using ex post aggregate prepay experience rooted in the assumed inefficiency of homeowners. But homeowners' behavior in the past presumably depended on a multitude of actual economic conditions (rate levels, credit conditions, demographics, unemployment rates, availability of housing, tax rates and breaks), and replicating these with aggregated statistics and market data allows subjective approximations to creep in. The hedge ratios for mortgages are thus impacted by the assumed idealizations in the model, and the model-driven hedging activity, if large enough (as in Fall 2002), can impact the prices of related securities and lead to a vicious feedback mechanism (need for duration led to buying of Treasuries, and falling yields led to higher prepayments and more need for duration).

People's behavior changes as a response to the economic environment, which models ought to but cannot accurately capture. How will a collapse of housing in California, were it to happen, affect risk premiums, given that homeowners have been buying homes using affordable mortgage products akin to credit cards? There are no real historical data on mortgage affordability products on which we can base statistical estimates. Experts can and will disagree.

I stress-test a number of models on their essential inputs for the same econometric variables, and find that models respond very differently to the same set of inputs. Two modelers working across the street from each other, with the same inputs and same training in finance, come up with very different models and different risk numbers for plain vanilla liquid mortgages.

PRICE SHOULD REFLECT VALUE

A quotation from none other than Adam Smith jumps to mind as a preview of the violation of the equivalence of price and value (Smith [1776]):

> The things that have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water; but it will purchase scarce any thing: scarce anything can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it.

It is relatively easy to create a model to justify the price of a security without reference to the economic environment, since in equilibrium demand and supply match. It is harder to create a model to justify the value of a security. Even for the most liquid securities, there is one price, but different values for different investors.

On September 26, 2005, Goldman, Sachs issued 30-year tax-exempt liberty bonds at a yield of approximately 4.59% when 30-year Treasuries were at 4.50%. For a taxable investor, this yield may or may not be absolutely attractive (grossed-up yield of almost 7%), given that we were at historically low levels in yields globally. But on a relative basis, versus Treasuries, this implies a higher-than 100% ratio, i.e., an implied term tax rate of 0% without adjusting for credit. Even after adjusting for credit, the ratio would be much higher than can be explained by tax rate uncertainty and was quite attractive.

Does the buyer of the muni who is short the tax option deserve so much higher a pre-tax yield? How much is the tax option really worth?

Since we cannot really hedge the tax risk, a relative-value statement on munis versus taxables requires us to incorporate economics in the model right from the beginning; taxes are an economic variable. Perhaps option pricing was more accurate before Black-Scholes came around, because it matched buyer and seller with an explicit recognition of their ability to take risk, and did not simply lean on the framework that we have come to believe in so deeply where all unwanted risks are assumed hedgeable.

This so-called muni puzzle has been around for decades, and in some cases it is blatantly in violation of arbitrage principles, i.e., that forward implied tax rates can go negative. To take advantage of the arbitrage opportunity, you would need to short a muni bond, which is notoriously hard, if not impossible, to do. You can, of course, lock in the potential of sure gains over some finite horizon by buying munis and shorting taxables according to some model-based hedge ratio, but then you are subject to volatile markets while you try to survive, hoping for arbitrage relation to enforce convergence to fair values.

The difference between price and value is captured by using risk-neutral versus risky distributions. In the world of credit, reduced-form models such as Duffie-Singleton are pricing models, while the Merton model is a valuation model, and in many practical cases you cannot simply transform from one to the other. Regressions that forecast ten-year bond yields based on macroeconomic variables are valuation models, while term structure models such as HJM, BGM, and BDT are pricing models. We cannot always connect the two approaches once they have been laid out—it is important to build the connections right from the beginning.

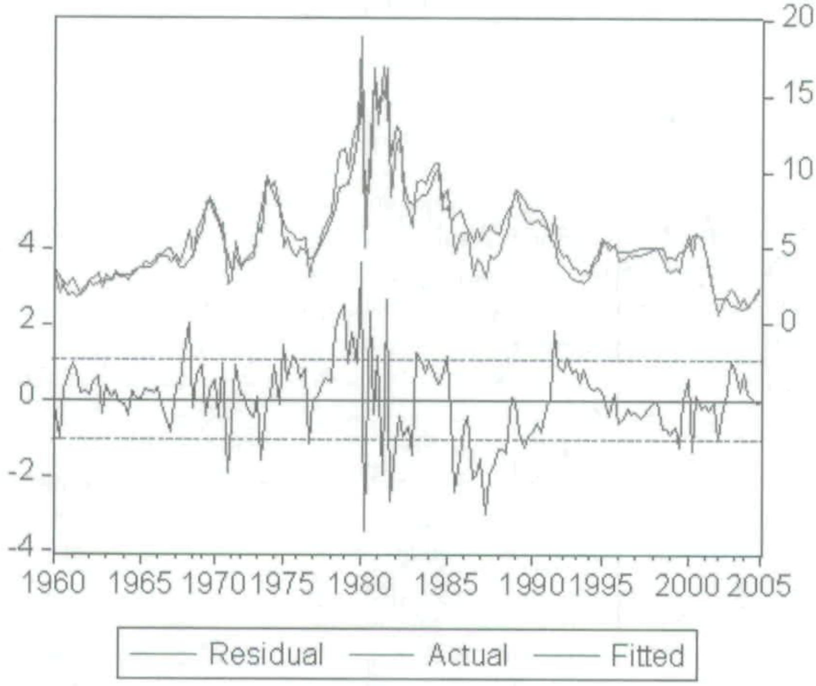
PRICES OF SECURITIES ARE IRRELEVANT FOR THE EVOLUTION OF THE STATE OF THE WORLD

Friction-free models assume that today's price of a traded security should not impact the state of the world in the future. But we saw in 2005 a great conundrum—the 10-year note yield remained very low by any standards, and led to a boom in housing and perhaps other long-term asset prices that take their cue from low funding rates.

It is hard to argue with the fact that current prices can impact tomorrow's economic environment, and tomorrow's environment will impact prices that follow . . . and so on. We have noted that the price of credit risk is lower now than it has been in several decades, and can be traced to the same common economic sources—easy liquidity, low risk aversion, and transparent policy.

Prices of securities do not simply indicate value in a transaction; they also have an important signaling effect,

EXHIBIT 9
Fed Funds Rate and z



Source: PIMCO.

which cannot be evaluated without putting the prices in the correct economic context. When the Treasury started to tinker with the long bond, the immediate impact was the loss of signaling ability of that part of the yield curve. There is a belief that purchases of Treasuries by foreigners have also impeded the information content of the yield curve, and the Fed and others have attempted to measure the magnitude of this impact (Bernanke, Reinhart, and Sack [2004]). Models that do not account for these nuances cannot work when the underlying assumptions undergo regime switches. Proper building blocks are essential.

So, can we properly put economics (back) into models? I propose one of many possible frameworks that has the advantage of being intuitive, easy to understand, and full of economic content. It does not abandon anything we know, but rather works with all that is good in the principles above and goes a step farther by giving it economic content.

I will focus on a model of the credit risk-free yield curve. Easy generalizations can be built for more complex securities (e.g., MBS, corporate bonds, and default swaps).

My approach is to build a well-motivated term structure model that is based on economics; estimate the parameters in the model using yield curve data in an arbitrage-free way; and then pick relevant economic variables that are related to the latent variables by regression. Exhibit 8 shows the key economic variables we will use—the inflation rate, real gross domestic product, current account deficit as a percent of GDP, and the difference in the ten-year and one-year Treasury rates as a proxy for the term premium.

A general form of a two-factor latent variable model is given by:

$$\begin{aligned} dx &= -\mu_x(x-\theta_x)dt + \sigma_x\, dw_x \\ dy &= -\mu_y\, y\,dt + \sigma_y\, dw_y \\ dz &= k(x+y-z)dt \end{aligned} \tag{2}$$

We can think of z as the instantaneous short rate and the equation for dz as a simplified Taylor rule, with results fitted in Exhibit 9. Since the change in z is related to the difference from the sum of x and y, it is tempting to identify these variables with the inflation and GDP gaps as in the Taylor rule and the difference equation for z as a form of the Taylor rule, and I will try to make this connection once time series for x and y are obtained by arbitrage-free fits to the yield curve.

The benefit of this simple specification is that we can bring in the machinery for solving for zero-coupon bond prices and yields using:

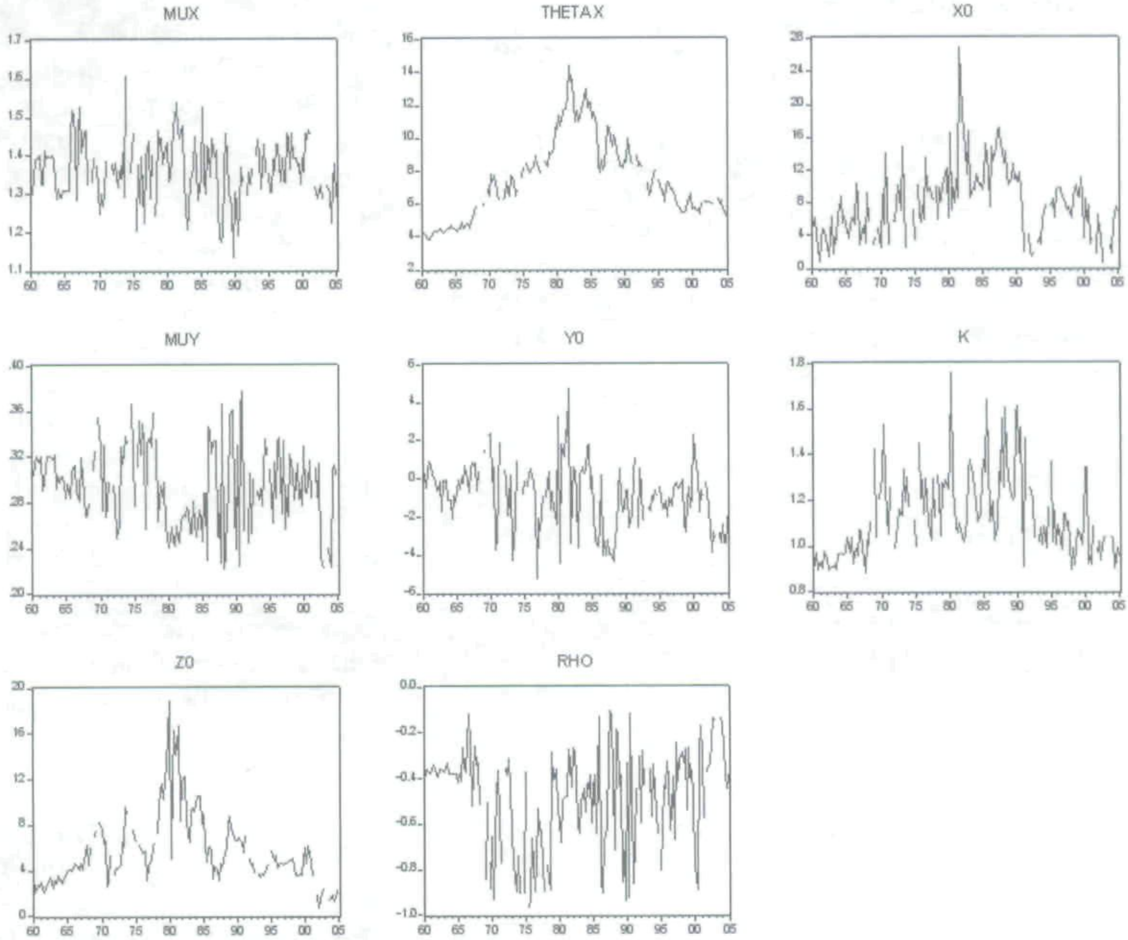
$$P(t,T) = E_t\left[e^{-\int_t^T z(\alpha)d\alpha}\right] = e^{-y(T-t)} \tag{3}$$

and where y is the yield at time t of a bond maturing at time T which can be solved in closed form. Once we have an analytical model for the discount factors, we can fit the model to the observed term structure every day (see Exhibit 10 for fits from 1960).

Finally, I estimate combinations of economic variables that correlate well with these estimated parameters. By construction, the structural shifts in the Taylor rule are transmitted across the yield curve via the latent factors x and y. Exhibit 11 shows that θ_x fits well with the inflation rate (plus some premium), and Exhibit 12 shows

EXHIBIT 10

Model Fits Using Only Treasuries as Input



Source: PIMCO.

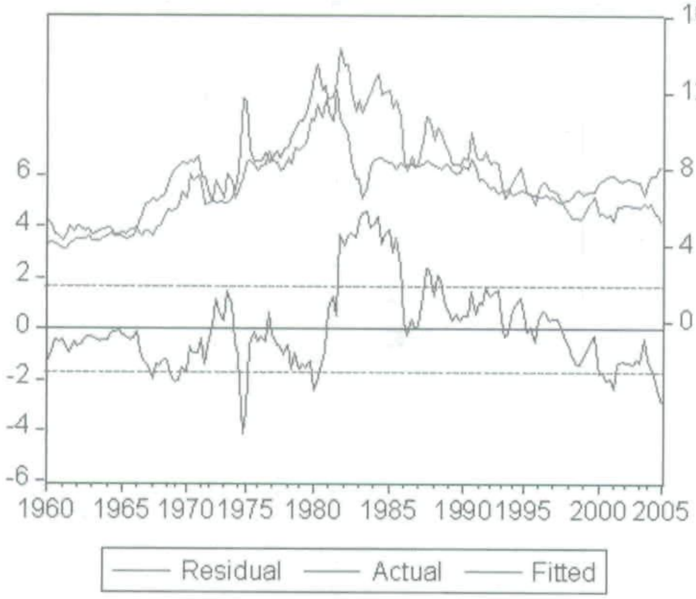
the dependence of the factor y on the slope of the yield curve (curve risk premium). Since the inflation risk factor in a portfolio is measured by the duration statistic, and the curve risk premium is measured by yield curve steepening or flattening exposure, we can also extract the macro risk embedded in security prices as derivatives of the price function with respect to the appropriate combination of latent factors.

We can also investigate the impact of the mushrooming U.S. current account deficit by adding the current account deficit (as a percent of GDP) to the regression for the long-term inflation rate θ_x to see if it has any explanatory power. As expected, and as shown in Exhibit 13, we find that the market-implied long-term inflation expectations have been kept much lower than they would have otherwise. For very long horizons, long-term inflation expectations could be almost two percentage points higher (difference of fitted residuals in the model with and without the current account deficit as an explanatory variable). We could improve our naive model in Equation (2) by carefully including the current account deficit indirectly in the response function.

Now that we see that economic variables have good explanatory power for the latent variables, there is no reason we should not build the relevant economics into the model right from the beginning; in other words, before fitting to the yield curve and security prices, we can constrain the ranges for the parameters and factors using our economic priors. This is effectively the methodology adopted by Ang and Piazzesi [2003]. We can also modify the policy response function to allow for new paradigms. In Bhansali and Wise [2005] and Dorsten,

EXHIBIT 11

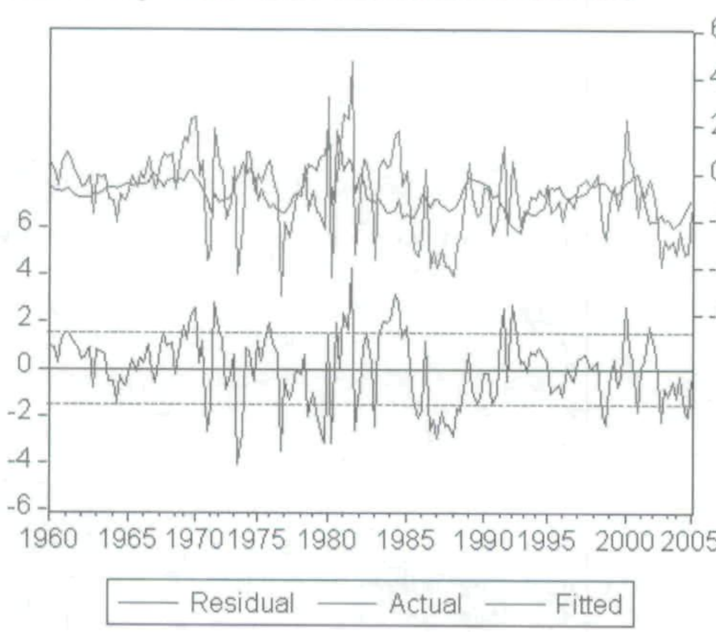
Model-Implied Long-Term Inflation Rate Regressed Against Actual Inflation



Regression equation θ_x = 4.868 + 0.619 (INFL yields r^2 of 0.46), with infl t-stats = 11.9.
Source: PIMCO.

EXHIBIT 12

Second Factor y versus Economic and Market Variables



Regression equation $y_0 = -0.14$–0.0422 (GDP-0.00579) (INFL-0.667) (10Y – 1Y). Only the slope factor 10Y – 1Y is significant. This indicates that the strongly mean-reverting second factor might be more correlated with risk premiums and preferences than with economic cycles. Regressions against market volatility indicators also show a high correlation of the second factor.
Source: PIMCO.

Bhansali, and Wise [2006], stochastic simulations of an asymmetric extension of the simple Taylor rule in Equation (1) explore the evolution and the economic content of the yield curve under deflation aversion.

We generate shapes of the yield curve that have an intuitive explanation in terms of new policy frameworks, but that might appear as conundrums in traditional frameworks. What is interesting is not just the ease of the bottom-up construction, but also the simple interpretation of the underlying economics.

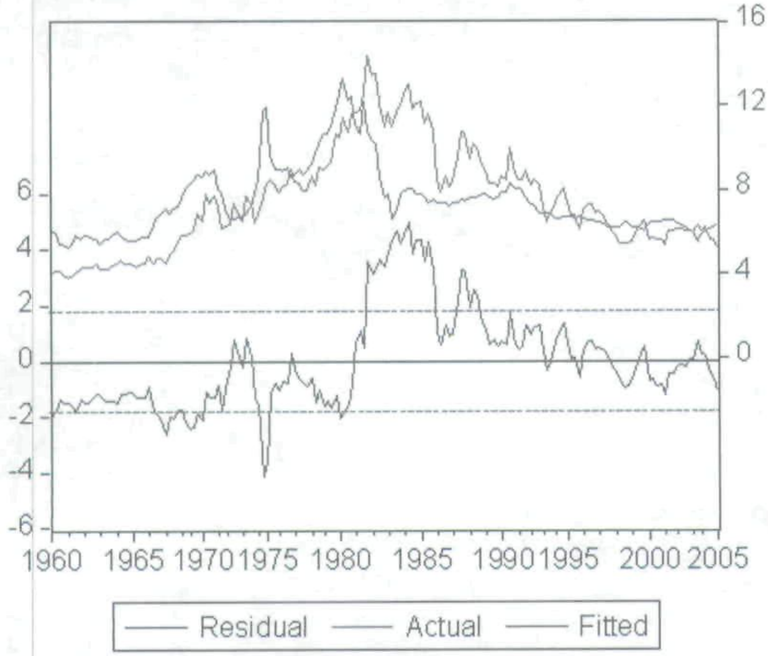
When we specified the model above, we used two sources of uncertainty plus the response function. We know from experience that yield curve fluctuations can be described by two factors plus a transient third factor related to curvature, so this approach is consistent with the literature. Fixed-income securities are influenced predominantly by inflation (and inflation expectations), Fed behavior, and risk premium. The latent factors are revealed by our regressions to be closely related to the inflation and GDP variables.

Another advantage of a model so specified is that we can estimate the effect of the yield curve on the macroeconomy, as well as the effect of the macroeconomy on the yield curve, by building in lags. A simple model such as the one described is computationally efficient for stress-testing real portfolios, but the biggest advantage is the model's usefulness in relative-value analysis without losing sight of the absolute content of the yield curve. In our work, we have extended the framework to incorporate credit risk, prepayment risk, and tax risk where stochastic intensity dynamics are correlated with macro factors.

Here I have systematically tried to put some economics back into models—at least a little bit of economics—but I am only scratching the surface. Much remains to be done. We see a resurgence of approaches where it is clear from the outset that economic variables can affect price, value, and risk, and price, value, and risk can affect economic

EXHIBIT 13

Long-Term Inflation Expectations Implied by Term Structure Model



Regressed against actual inflation rates and current account deficit. Regression equation $\theta_x = 4.13 + 0.687$ (INFL-0.387) (CURPGDP), where t-stats on inflation rate equal 13, and on current account deficit equal 5, so both variables are significant. Regression $r^2 = 0.54$.
Source: PIMCO.

variables—something that as participants in the markets we already know and practice. This understanding is bringing to the forefront of the modeling profession what we already know intuitively: The economic state of the world *does* matter for investment and portfolio analysis.

ENDNOTE

*Hurricane Hazel hit southwestern Ontario on October 14, 1954.

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